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The Gross-Neveu Model from String Theory

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We study an intersecting D-brane model which at low energies describes $(1+1)$ -dimensional chiral fermions localized at defects on a stack of N_c $D4$ -branes. Fermions at different defects interact via exchange of massless $(4+1)$ -dimensional fields. At weak coupling this interaction gives rise to the Gross-Neveu (GN) model and can be studied using field theoretic techniques. At strong coupling one can describe the system in terms of probe branes propagating in a curved background in string theory. The chiral symmetry is dynamically broken at zero temperature and is restored above a critical temperature T_c which depends on the coupling. The phase transition at T_c is first order at strong coupling and second order at weak coupling.

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1. Introduction

In this paper we continue the investigation of dynamical symmetry breaking in intersecting D-brane models. The construction of these models starts with N_c BPS Dp -branes, which we will refer to as color branes. The low energy theory on these branes is $(p+1)$ -dimensional $U(N_c)$ super Yang-Mills (SYM) with sixteen supercharges. For $p > 3$, the Yang-Mills coupling g_{p+1} scales like a positive power of length, and the SYM degrees of freedom become free in the infrared.

To get non-trivial infrared dynamics we add additional D-branes, which we will refer to as flavor branes. The flavor branes intersect the color ones on subspaces of the $(p+1)$ -dimensional worldvolume of the latter. Open strings stretched between the color and flavor branes give rise to light fermions (and sometimes bosons as well) that are localized at the intersections. At low energies, fermions at different intersections interact via exchange of $(p+1)$ -dimensional massless fields living on the color branes. In some cases, these interactions lead to non-trivial infrared effects.

When the $(p+1)$ -dimensional $U(N_c)$ SYM theory is weakly coupled in the infrared, the effective interaction between fermions living at different defects becomes weaker as the distance between the defects, L , increases. The leading long distance dynamics can be studied in an approximation where one includes the effective four-Fermi interaction that arises from exchange of a single $(p+1)$ -dimensional massless field between two fermions, but neglects all the higher order exchange processes. Such processes are suppressed since the gauge coupling g_{p+1} goes to zero at long distances.

A nice thing about this approximation is that it leads to a theory that is tractable at large N_c . Indeed, since the only dynamical degrees of freedom are the fermions, which transform in the vector representation of $U(N_c)$, the low-energy dynamics is described by a *vector model*, and can typically be solved using familiar large N_c field theoretic techniques.

As L decreases, the interactions between the fermions and those between the fermions and the color degrees of freedom become stronger. For small L the dynamics can in some cases be analyzed using holography. In this limit the N_c color branes can be replaced by their near-horizon geometry, and the interactions among the fermions are encoded in the dynamics of the flavor branes in the resulting curved spacetime.

An example of an intersecting brane system of the kind described above is the $D4 - D8 - \overline{D8}$ system, which was studied from the present perspective in [1], following some closely related earlier work [2,3]. In that model the field theoretic analysis of the weak

coupling regime is complicated by the fact that the effective four-Fermi interaction due to single gluon exchange is long range – it is given by a non-local generalization of the Nambu-Jona-Lasinio model [4]. This leads to some interesting subtleties in the above considerations to which we hope to return elsewhere.

The main purpose of this paper is to analyze an intersecting brane system which does not suffer from such subtleties. Our discussion is organized as follows.

In section 2 we describe a brane configuration consisting of N_c color $D4$ -branes and N_f flavor $D6$ and $\overline{D6}$ -branes in type IIA string theory, intersecting on a $(1+1)$ -dimensional spacetime. The $D4 - D6$ and $D4 - \overline{D6}$ intersections are separated by a distance L in the \mathbb{R}^3 along the fourbranes and transverse to the sixbranes. At the two intersections one finds chiral left and right-moving fermions, respectively. The low-energy dynamics of these fermions is governed by exchange of massless fields that live on the $D4$ -branes, which are described by a five-dimensional gauge theory with 't Hooft coupling λ (which has units of length). The strength of the interaction among the fermions is determined by the value of the dimensionless parameter λ/L .

In section 3 we study the system at weak coupling, where one can (to leading order in λ/L) restrict to the single gluon exchange approximation. The resulting four-Fermi interaction between the left and right-moving fermions is described by the Gross-Neveu model [5], with a specific UV cutoff. This model is known to break chiral symmetry, and to generate a mass scale. We rederive the chiral symmetry breaking from our perspective, and show that the dynamically generated mass of the fermions is much smaller than $1/L$. This allows us to take a decoupling limit in which the intersecting brane model reduces precisely to the GN model.

In section 4 we discuss the system at strong coupling, *i.e.* for $\lambda \gg L$. In this regime the fermions and massless fields on the $D4$ -branes are strongly interacting, but there is still a weakly coupled description of the physics. It involves replacing the color $D4$ -branes by their near-horizon geometry, and studying the flavor $D6$ -branes as probes in this geometry. As in [1], there are two possible shapes that the flavor branes can take. One is the original separated parallel $D6$ and $\overline{D6}$ -branes. The other is a single connected brane which looks like the original $D6$ and $\overline{D6}$ -branes connected by a wormhole whose width increases with λ/L . We find that (as in [1]) the curved, connected brane has lower energy than the pair of separated straight ones. This implies that the chiral symmetry acting on the left and right-moving fermions is dynamically broken in this regime as well. The energy scale associated

with the breaking increases with the coupling λ/L . We also comment on the spectrum of the model in the strong coupling regime and its relation to that of the Gross-Neveu model.

In section 5 we discuss the finite temperature behavior of the model. In the weak coupling regime the chiral condensate decreases as the temperature increases and vanishes above a certain critical temperature T_c . Thus, the chiral symmetry is restored in a second order phase transition at $T = T_c$. At strong coupling, the (free) energy difference between the connected curved and separated straight branes decreases as the temperature increases, and beyond a certain critical value of the temperature the straight branes have lower free energy. The chiral symmetry restoration transition is strongly first order in this limit.

We end in section 6 with conclusions and a discussion of some open problems.

2. $D4 - D6 - \overline{D6}$ brane configuration and its low-energy dynamics

The brane configuration that we will consider is depicted in figure 1. It contains the following D -branes:

$$\begin{array}{cccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
 D4 : & x & x & x & x & x & & & & & \\
 D6, \overline{D6} : & x & x & & & & & x & x & x & x & x
 \end{array} \tag{2.1}$$

N_c color $D4$ -branes stretched in (01234) are located at the origin in (56789). N_f flavor $D6$ and $\overline{D6}$ -branes are stretched in (0156789) and separated by a distance L in (234). Without loss of generality we can take the separation between the flavor branes and anti-branes to be in the x^4 direction. The color and flavor branes intersect on the $(1+1)$ -dimensional spacetime labeled by (01). We will be primarily interested in the low-energy dynamics at the intersection.

The $(9+1)$ -dimensional Lorentz symmetry is broken by the branes (2.1) as follows:

$$SO(1, 9) \rightarrow SO(1, 1)_{01} \times SO(2)_{23} \times SO(5)_{56789} . \tag{2.2}$$

Each of the two intersections preserves a chiral supersymmetry. The $D4 - D6$ intersection preserves eight right-handed supercharges under $SO(1, 1)$, while the $D4 - \overline{D6}$ intersection preserves eight left-handed supercharges. The full system breaks all supersymmetry.

We will study the system of figure 1 in the limit $N_c \rightarrow \infty$, $g_s \rightarrow 0$, with $g_s N_c$ and N_f held fixed and $L \gg l_s$. In this limit the coupling of $6 - 6$, $\overline{6} - \overline{6}$ and $6 - \overline{6}$ strings goes to

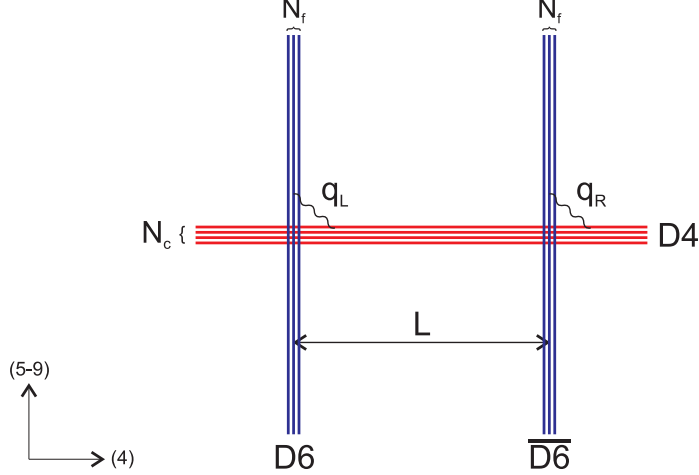


Fig. 1: The configuration of N_c $D4$ -branes and N_f $D6$ and $\overline{D6}$ -branes which reduces at weak coupling to the Gross-Neveu model.

zero and they become non-dynamical external sources. The gauge symmetry on the $D6$ and $\overline{D6}$ -branes,

$$U(N_f)_L \times U(N_f)_R , \quad (2.3)$$

gives rise to a global symmetry of the $(1+1)$ -dimensional theory at the intersection.

The low-energy degrees of freedom are low lying $4-4$ strings stretched between color branes, as well as $4-6$ and $4-\overline{6}$ strings stretched between color branes and flavor branes and anti-branes, respectively. The low energy dynamics in the $4-4$ sector is described by dimensional reduction of $(9+1)$ -dimensional $N=1$ SYM with gauge group $U(N_c)$ to $4+1$ dimensions. In the $4-6$ sector one finds left-moving fermions q_L transforming in the $(\overline{N}_f, 1)$ of the global symmetry (2.3). $4-\overline{6}$ strings give right-moving fermions q_R which transform as $(1, \overline{N}_f)$. Both q_L and q_R transform in the fundamental (N_c) representation of the $U(N_c)$ gauge symmetry on the $D4$ -branes. Note in particular that the $U(N_f)_L$ symmetry in (2.3) acts only on the left-moving fermions, while $U(N_f)_R$ acts only on the right-moving ones. Thus, (2.3) is a symmetry of the Lagrangian, but we will see later that it is broken to the diagonal $U(N_f)$ by the dynamics.

The 't Hooft coupling of the $(4+1)$ -dimensional gauge theory on the $D4$ -branes, λ , has units of length. The theory is weakly coupled for distances much larger than λ and strongly coupled otherwise. In order to define it for all distance scales one must supply a UV completion. String theory does that by embedding it in a six-dimensional CFT, the $(2,0)$ theory. The $(4+1)$ -dimensional gauge theory is the low-energy limit of the $(2,0)$ theory compactified on a circle whose radius is proportional to $g_s l_s$.

The fermions arising from $4 - 6$ and $4 - \bar{6}$ strings live on codimension three defects in the $(4 + 1)$ -dimensional gauge theory. Their separation, L , introduces a second distance scale into the dynamics. One can think of the defects as probing the dynamics of the gauge theory (or, more properly, of the $(2, 0)$ theory compactified on S^1) at the distance scale L .

For $L \gg \lambda$ the theory on the fourbranes is weakly coupled. Naively, the $(4 + 1)$ -dimensional fields that transform in the adjoint of $SU(N_c)$ decouple in this limit, just like the corresponding modes on the sixbranes. This is not quite correct, since exchange of $(4 + 1)$ -dimensional massless modes leads to a weak attractive interaction between the left and right-handed fermions for any finite λ . As we will see, this interaction leads to chiral symmetry breaking and has to be kept in the low energy description.

The effective action that describes the dynamics in this limit contains the fermions q_L , q_R , and five-dimensional gauge fields on the $D4$ -branes, A_M , $M = 0, 1, 2, 3, 4$,

$$\mathcal{S} = \int d^5x \left[-\frac{1}{4g_5^2} F_{MN}^2 + \delta^3(\vec{x} - \vec{x}_1) q_L^\dagger \bar{\sigma}^\mu (i\partial_\mu + A_\mu) q_L + \delta^3(\vec{x} - \vec{x}_2) q_R^\dagger \sigma^\mu (i\partial_\mu + A_\mu) q_R \right], \quad (2.4)$$

where the notation is as follows. The $(4 + 1)$ -dimensional worldvolume of the color $D4$ -branes is parametrized by $\{x^M\} = (x^0, x^1, \vec{x})$. The flavor branes and anti-branes are placed at $\vec{x} = \vec{x}_1$ and \vec{x}_2 , respectively. As mentioned above, we can take \vec{x}_1, \vec{x}_2 to lie on the x^4 axis, with $|\vec{x}_1 - \vec{x}_2| = L$. The indices μ in (2.4) run over the range $\mu = 0, 1$ or $+, -$. q_L, q_R are complex one component spinors. The corresponding one-dimensional Dirac matrices are $\bar{\sigma}^\mu = \delta_+^\mu$, $\sigma^\mu = \delta_-^\mu$.

We will define the 't Hooft coupling of the $(4 + 1)$ -dimensional gauge theory on the color branes in terms of g_5 (2.4) by¹

$$\lambda = g_s N_c = \frac{g_5^2 N_c}{(2\pi)^2}. \quad (2.5)$$

As mentioned above, λ is a length scale. In the next section we will use the Lagrangian (2.4) to analyze the interaction between the fermions q_L and q_R for $\lambda \ll L$. In the opposite limit of large λ/L we cannot use (2.4), both because we have to add to it the other massless fields living on the $D4$ -branes, and because in that regime the dynamics of these modes is no longer described by weakly coupled field theory in $4 + 1$ dimensions. Nevertheless, one can still efficiently study the theory in this limit. This will be discussed in section 4.

¹ Here and below we often set $\alpha' = 1$.

3. Weak coupling analysis

In order to study the dynamics of the fermions q_L, q_R at weak coupling, it is convenient to integrate out the five-dimensional fields living on the color $D4$ -branes. To leading order in λ/L this can be done by taking into account the interaction among the fermions due to single gluon exchange, using the action (2.4). The calculation is similar to that of [1] and leads to

$$\begin{aligned} \mathcal{S}_{\text{eff}} = & i \int d^2x \left(q_L^\dagger \bar{\sigma}^\mu \partial_\mu q_L + q_R^\dagger \sigma^\mu \partial_\mu q_R \right) \\ & + \frac{g_5^2}{4\pi^2} \int d^2x d^2y G(x-y, L) \left(q_L^\dagger(x) \cdot q_R(y) \right) \left(q_R^\dagger(y) \cdot q_L(x) \right) \end{aligned} \quad (3.1)$$

As in [1], $q_L^\dagger(x) \cdot q_R(y)$ is a global color singlet (hence the dot) and transforms in the (N_f, \bar{N}_f) of $U(N_f)_L \times U(N_f)_R$. $G(x, L)$ is the five-dimensional massless propagator over a distance x in the directions (01) and L in the directions (234),

$$G(x, L) = \frac{1}{(x^2 + L^2)^{\frac{3}{2}}} . \quad (3.2)$$

In addition, there are self interactions between fermions at a given intersection (the analog of eq. (3.2) in [1]), but these are small for distances much larger than λ and can be neglected. We also Wick rotated to Euclidean space, which is convenient for analyzing the vacuum structure.

Many properties of the solution of the interacting quantum field theory corresponding to the action (3.1) can be anticipated as follows. The main complication in (3.1) is the non-local four Fermi interaction on the second line. If the function $G(x-y, L)$ was proportional to $\delta^2(x-y)$, (3.1) would reduce to the Gross-Neveu model [5], an asymptotically free local quantum field theory which is exactly solvable in the large N_c limit.

The actual Green function, (3.2) can be thought of as a δ -function smeared over a region of size L . Using the fact that

$$\int d^2x G(x, L) = \frac{2\pi}{L} , \quad (3.3)$$

we can, at length scales large compared to L , replace (3.1) by the local action

$$\mathcal{S}_{\text{eff}} = \int d^2x \left[i q_L^\dagger \bar{\sigma}^\mu \partial_\mu q_L + i q_R^\dagger \sigma^\mu \partial_\mu q_R + \frac{1}{N_c} \frac{2\pi\lambda}{L} \left(q_L^\dagger(x) \cdot q_R(x) \right) \left(q_R^\dagger(x) \cdot q_L(x) \right) \right] \quad (3.4)$$

with a UV cutoff $\Lambda \simeq \frac{1}{L}$. Comparing to [5] we see that the Gross-Neveu coupling λ_{gn} is given in terms of our parameters by

$$\lambda_{gn} = \frac{2\pi\lambda}{L} . \quad (3.5)$$

In [5] it was found that at large N_c the model (3.4) dynamically generates a fermion mass which goes like

$$M_F \simeq \mu e^{-\frac{2\pi}{\lambda_{gn}}} \quad (3.6)$$

where μ is an arbitrary renormalization scale and λ_{gn} the coupling at that scale. Since our model reduces in a particular limit (which will be made more precise below) to the Gross-Neveu model, it is natural to expect that it exhibits similar behavior. We next verify this by a more detailed analysis following closely that of [1].

To solve the theory (3.1) for large N_c it is convenient to introduce a bilocal field $T(x, y)$ which is a singlet of global $U(N_c)$ and transforms as (N_f, \overline{N}_f) under the symmetry (2.3). The action (3.1) can be rewritten as

$$\begin{aligned} \mathcal{S}_{\text{eff}} = & i \int d^2x \left(q_L^\dagger \overline{\sigma}^\mu \partial_\mu q_L + q_R^\dagger \sigma^\mu \partial_\mu q_R \right) \\ & + \int d^2x d^2y \left[-\frac{N_c}{\lambda} \frac{T(x, y) \overline{T}(y, x)}{G(x - y, L)} + \overline{T}(y, x) q_L^\dagger(x) \cdot q_R(y) + T(x, y) q_R^\dagger(y) \cdot q_L(x) \right] . \end{aligned} \quad (3.7)$$

Integrating out $T(x, y)$ using its equation of motion,

$$T(x, y) = \frac{\lambda}{N_c} G(x - y, L) q_L^\dagger(x) \cdot q_R(y) , \quad (3.8)$$

one recovers the original action (3.1). To solve the large N_c theory one instead integrates out the fermions and obtains an effective action for T , which becomes classical in the large N_c limit. As in [1], using translation invariance of the vacuum, it is enough to compute this effective action for $T(x, y) = T(|x - y|)$. Dividing by N_c and the volume of spacetime we get the effective potential

$$V_{\text{eff}} = \frac{1}{\lambda} \int d^2x \frac{T(x) \overline{T}(x)}{G(x, L)} - \int \frac{d^2k}{(2\pi)^2} \ln \left(1 + \frac{T(k) \overline{T}(k)}{k^2} \right) \quad (3.9)$$

where $T(k)$ is the Fourier transform of $T(x)$. Varying (3.9) with respect to $\overline{T}(k)$ leads to the gap equation

$$\frac{1}{\lambda} \int d^2x \frac{T(x)}{G(x, L)} e^{-ik \cdot x} = \frac{T(k)}{k^2 + T(k) \overline{T}(k)} . \quad (3.10)$$

The solution of (3.10) gives the expectation value of the chiral condensate $\langle q_L^\dagger(x) \cdot q_R(y) \rangle$ via (3.8). This equation has a trivial solution $T = 0$, but we will next see that there is a non-trivial solution as well. As shown in [1], any well behaved non-trivial solution of (3.10) has a negative value of V_{eff} ; hence the vacuum of the theory breaks chiral symmetry.

To solve the gap equation for small λ/L it is convenient to define the field $f(x, y)$ by

$$T(x, y) = \frac{L}{2\pi} G(x - y, L) f(x, y) . \quad (3.11)$$

Using (3.8) we see that the expectation value of $f(x, y)$ is proportional to the condensate $\langle q_L^\dagger(x) \cdot q_R(y) \rangle$,

$$f(x - y) \equiv \langle f(x, y) \rangle = \frac{\lambda_{gn}}{N_c} \langle q_L^\dagger(x) \cdot q_R(y) \rangle . \quad (3.12)$$

In particular, the condensate at coincident points, $\langle q_L^\dagger(x) \cdot q_R(x) \rangle$ is proportional to $f(0)$, which we will denote by m_f . We will determine m_f as part of the solution of the gap equation. We will assume (and justify later) that $f(x)$ is approximately constant on the scale L . Since the function $G(x, L)$ (3.2) goes rapidly to zero for $x > L$ (and can be thought of as a smeared δ -function) this implies that

$$T(k) = \frac{L}{2\pi} \int d^2x G(x, L) f(x) e^{-ik \cdot x} \simeq m_f e^{-kL} . \quad (3.13)$$

Substituting (3.11) and (3.13) into the gap equation (3.10) gives the Fourier transform of $f(x)$:

$$\tilde{f}(k) = \lambda_{gn} \frac{m_f e^{-|k|L}}{k^2 + m_f^2 e^{-2|k|L}} . \quad (3.14)$$

We can now determine the mass parameter m_f by evaluating $f(0)$ in terms of its Fourier transform:

$$m_f = f(0) = \lambda_{gn} \int \frac{d^2k}{(2\pi)^2} \frac{m_f e^{-|k|L}}{k^2 + m_f^2 e^{-2|k|L}} . \quad (3.15)$$

Dividing by m_f and estimating the momentum integral for $m_f L \ll 1$ we find

$$1 \simeq \frac{\lambda_{gn}}{2\pi} \log(\Lambda/m_f) \quad (3.16)$$

where $\Lambda \sim 1/L$ is the UV cutoff of the effective GN model (3.4). Note that the dynamically generated mass scale

$$m_f \simeq \Lambda e^{-2\pi/\lambda_{gn}} \quad (3.17)$$

is much smaller than the cutoff Λ for small GN coupling.

To complete the discussion, we need to show that the resulting $f(x)$ is slowly varying on the scale L (to justify (3.13)). Fourier transforming (3.14) we find the following behavior. For $x \ll L$,

$$f(x) \simeq f(0) = m_f . \quad (3.18)$$

For $L \ll x \ll 1/m_f$,

$$f(x) \simeq m_f \frac{\ln(xm_f)}{\ln(Lm_f)} , \quad (3.19)$$

while for $x \gg 1/m_f$, f goes exponentially to zero.

Although $f(x)$ for $x > L$ is not constant, it is very slowly varying at weak coupling. Indeed,

$$\frac{f(0) - f(x)|_{x \gg L}}{f(0)} \simeq 1 - \frac{\ln(xm_f)}{\ln(Lm_f)} \simeq \frac{\lambda_{gn}}{2\pi} \ln \frac{x}{L} . \quad (3.20)$$

The logarithmic variation (3.19), (3.20) becomes important only on scales for which $\frac{\lambda_{gn}}{2\pi} \ln \frac{x}{L} \simeq 1$, *i.e.* $x \simeq 1/m_f$. We conclude that $f(x)$ (3.11) varies on the scale $1/m_f \gg L$.

The same conclusion can be reached by examining the momentum space expression (3.14). The Fourier transform $\tilde{f}(k)$ contains two momentum scales, m_f and $1/L$. However, most of its support is on the scale m_f . At $k = 1/L$ it has been reduced by a factor $e^{-2L/\lambda}$ from its value at the origin. This means that the scale of variation of $f(x)$ is $1/m_f$.

The solution for f gives the following position dependent condensate:

$$\langle q_L^\dagger(x) \cdot q_R(0) \rangle = N_c m_f \int_{|k| < \Lambda} \frac{d^2 k}{(2\pi)^2} \frac{e^{ik \cdot x}}{k^2 + m_f^2} . \quad (3.21)$$

This is nothing but the massive propagator for a fermion of mass m_f . Thus, we conclude that the intersecting D-brane model generates dynamically a mass m_f (3.17) for the fermions living at the intersections of the color and flavor branes. This mass is non-perturbative in the Gross-Neveu coupling (3.5). Comparing (3.6) to (3.17) we see that it exhibits the same dependence on the coupling as in the field theoretic analysis of [5].

One can take a decoupling limit in which the physics of the brane configuration reduces precisely to the Gross-Neveu model. To do that one takes $\lambda/L \rightarrow 0$ and focuses on energy scales of order m_f . This sends the ratio of the mass of the fermions m_f (3.17) and the UV cutoff scale $1/L$ to zero, and leads to a model that is precisely equivalent to that of [5]. The model with finite λ/L can be thought of as a version of Gross-Neveu with a finite UV cutoff.

In the above discussion we took the renormalization scale of the theory to be the UV cutoff $\Lambda \sim 1/L$. λ_{gn} (3.5) is the coupling at that scale. One can choose some other

renormalization scale $\mu < \Lambda$ and define the coupling λ_{gn} at that scale. The renormalization group guarantees that Green functions expressed in terms of the physical fermion mass are insensitive to such changes.

The Gross-Neveu model (3.4), which describes the dynamics of q_L and q_R at weak coupling, was studied extensively in the past, and much is known about it. We next review some of its pertinent features. We first address the question of its extreme low-energy limit. This can be analyzed as follows (see *e.g.* [6,7]). The CFT of free massless fermions q_L, q_R can be bosonized in terms of a WZW model for the group

$$SU(N_c)_{N_f} \times SU(N_f)_{N_c} \times U(1) . \quad (3.22)$$

The subscripts in (3.22) denote the levels of the corresponding current algebras. One can write the left-moving currents corresponding to (3.22) in terms of the fermions q_L as follows:

$$\begin{aligned} U(1) : \quad J &= (q_L^*)^\alpha_i (q_L)_\alpha^i ; \\ SU(N_c)_{N_f} : \quad J_\beta^\alpha &= (q_L^*)^\alpha_i (q_L)_\beta^i - \frac{1}{N_c} \delta_\beta^\alpha J ; \\ SU(N_f)_{N_c} : \quad J_j^i &= (q_L^*)^\alpha_j (q_L)_\alpha^i - \frac{1}{N_f} \delta_j^i J . \end{aligned} \quad (3.23)$$

Here $\alpha, \beta = 1, \dots, N_c$ are color indices and $i, j = 1, \dots, N_f$ are flavor ones. A similar decomposition holds for the right-moving fermions. The currents on each line of (3.23) satisfy standard affine Lie algebra OPE's, and commute with those on the other two lines. The $U(1)$ current on the first line describes a massless free field. The rest of the low-energy theory (3.22) is an interacting WZW model (for generic N_f, N_c).

The four-Fermi interaction (3.4) can be written in terms of the currents (3.23), as a linear combination of an abelian Thirring interaction for the $U(1)$ factor, and a non-abelian Thirring interaction corresponding to $SU(N_c)$. The abelian Thirring interaction

$$\mathcal{L}_1 = \lambda_1 J \bar{J} \quad (3.24)$$

corresponds to changing the radius of the decoupled scalar field associated with the $U(1)$ factor in (3.22). Thus, its effects are easy to analyze. The non-abelian Thirring interaction associated with $SU(N_c)$,

$$\mathcal{L}_2 = \lambda_2 J_\beta^\alpha \bar{J}_\alpha^\beta , \quad (3.25)$$

leads to non-trivial long distance dynamics. The coupling λ_2 is asymptotically free. It grows in the infrared, where the $SU(N_c)_{N_f}$ degrees of freedom become massive and decouple. The extreme infrared theory is a coset model. It is obtained from the UV theory,

which contains the current algebra (3.23), by gauging the $SU(N_c)_{N_f}$ subalgebra. This eliminates the corresponding factor in (3.22) and leads to a WZW model for

$$SU(N_f)_{N_c} \times U(1) . \quad (3.26)$$

One can think of this model as describing the infrared fluctuations of the order parameters associated with chiral symmetry breaking.

As discussed in [6,7] these fluctuations effectively restore the chiral symmetry at finite N_c . For example, the two point function

$$\langle q_L^\dagger(x) \cdot q_R(x) q_R^\dagger(y) \cdot q_L(y) \rangle \quad (3.27)$$

does not in fact approach a constant at large separation, as would be expected from (3.21), but rather decays like $|x - y|^{-\frac{1}{N_c}}$. Thus, at infinite N_c the symmetry is broken, but at finite N_c it is restored at sufficiently long distances. As N_c increases, the range of distances for which the symmetry appears to be broken increases exponentially with N_c , and the large N_c analysis becomes a better and better approximation to the dynamics.

It will be useful below to determine the location of the massless modes (3.26) in the extra dimensions. This can be done following [8] by integrating out the fermions q_L , q_R and studying the normalizable modes of the resulting theory of gauge fields. To determine the fate of the $SU(N_f)$ factor² in the WZW model (3.26) it is enough to consider a single intersection, say that of the N_c $D4$ -branes and N_f $D6$ -branes. Since we are interested in the low-energy dynamics, we can restrict to s-waves on the four-spheres in the (56789) directions and write an effective three-dimensional Lagrangian for the three-dimensional $SU(N_f)$ gauge field with components³ (A_+, A_-, A_u) . That Lagrangian contains three types of terms,

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{inflow}} . \quad (3.28)$$

The first is the standard kinetic term, which has the form

$$\mathcal{L}_{\text{kin}} = \frac{1}{g_7^2} u^4 \text{Tr} \left[\frac{1}{2} (F_{+-})^2 - F_{u+} F_{u-} \right] . \quad (3.29)$$

The second comes from integrating out the fermions at the intersection. It is localized at $u = 0$ and has the form

$$\mathcal{L}_{\text{ferm}} = \delta(u) N_c \Gamma(A(0)) . \quad (3.30)$$

² The $U(1)$ factor in (3.26) can also be treated as in [8].

³ u is the radial direction in (56789).

Here $\Gamma(A)$ is the chiral Polyakov-Wiegmann action [9] corresponding to $SU(N_f)$, and $A(0)$ stands for the two-dimensional gauge field at the intersection, $A_{\pm}(x^+, x^-, 0)$.

The third term in (3.28) is a Chern-Simons term, which arises as follows. On the worldvolume of the $D6$ -branes there is a coupling of the $SU(N_f)$ gauge field A to the RR four-form field strength⁴ H_4 :

$$\frac{1}{8\pi^2} \int H_4 \wedge \omega_3(A) \quad (3.31)$$

where $\omega_3(A)$ is the Chern-Simons form for $SU(N_f)$,

$$\omega_3(A) = \text{Tr} \left(A \wedge F + \frac{2}{3} A \wedge A \wedge A \right) . \quad (3.32)$$

One way of deducing the presence of the term (3.31) and its coefficient is to require the cancellation of the $SU(N_f)$ anomaly of the fermions living on the $D4 - D6$ intersection by anomaly inflow [10].

The N_c $D4$ -branes (2.1) are localized in the directions (56789) and are magnetically charged under H_4 . Therefore, there are N_c units of H_4 flux going through the four-sphere at fixed u ,

$$\int_{S^4} H_4 = 2\pi N_c . \quad (3.33)$$

Integrating (3.31) over the sphere leads to a Chern-Simons term in the Lagrangian (3.28):

$$\mathcal{L}_{\text{inflow}} = \frac{N_c}{4\pi} \omega_3(A) . \quad (3.34)$$

The resulting Lagrangian (3.28) is very similar to one that was recently studied in [8]. An analysis similar to that of [8] shows that the $SU(N_f)$ degrees of freedom, which classically live at the intersection $u = 0$, are displaced to $u \simeq \lambda^{\frac{1}{3}}$. This displacement is small for weak coupling, but it has interesting implications at strong coupling, as in [8].

Note that while in the above discussion we analyzed the position of the chiral $SU(N_f)$ modes associated with a given intersection, the answer is the same for the system we are interested in, which has two intersections. As explained above, in that system there is a Thirring interaction for the $SU(N_c)$ and $U(1)$ factors in (3.22). Since the $SU(N_f)$ currents commute with these perturbations, they are not influenced by them. This decoupling between the massless sector and a non-trivial massive sector of a two-dimensional field theory is a rather general phenomenon; see *e.g.* [11] for a discussion.

⁴ We normalize the RR forms and Chern-Simons couplings as in [2].

So far we focused on the properties of the massless modes associated with the brane configuration of section 2. The massive spectrum of the non-abelian Thirring model (3.25) is also known, since the model is integrable [12-15]. The masses of single particle states are given by

$$M_n = m_f \frac{\sin \frac{\pi n}{N_c}}{\sin \frac{\pi}{N_c}} . \quad (3.35)$$

The integer n runs over the range $n = 1, 2, \dots, N_c - 1$. The $n = 1$ states are the original fermions. States with higher n can be thought of as bound states of n fermions.

4. Strong coupling analysis

Bringing the $D6$ -branes closer or increasing $g_s N_c$ increases λ/L and takes us to the strong coupling regime in which the approximations used in the previous section break down. For $L \ll \lambda$ we can employ a dual description of the model by studying the DBI action for $D6$ -branes in the near-horizon geometry of the N_c $D4$ -branes. The discussion follows closely that of [1,2].

4.1. Chiral Symmetry Breaking

The near-horizon metric and dilaton of the $D4$ -branes are given by

$$\begin{aligned} ds^2 &= \left(\frac{U}{R}\right)^{\frac{3}{2}} [\eta_{\mu\nu} dx^\mu dx^\nu - (dx^4)^2] - \left(\frac{U}{R}\right)^{-\frac{3}{2}} (dU^2 + U^2 d\Omega_4^2) , \\ e^\Phi &= g_s \left(\frac{U}{R}\right)^{\frac{3}{4}} , \end{aligned} \quad (4.1)$$

where we set $\alpha' = 1$, Ω_4 labels the angular directions in (56789), and the parameter R is given by

$$R^3 = \pi g_s N_c = \frac{g_5^2}{4\pi} N_c = \pi \lambda . \quad (4.2)$$

Recall that the $(4+1)$ -dimensional 't Hooft coupling λ has units of length. There is also a non-zero RR four-form field strength H_4 with N_c units of flux around the S^4 in (4.1).

The $D6$ -brane⁵ is stretched in the directions (01), wraps the four-sphere labeled by Ω_4 , and forms a curve $U = U(x^4)$ in the (U, x^4) plane. The induced metric on it is

$$d\tilde{s}^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} [(dx^0)^2 - (dx^1)^2] - \left(\frac{U}{R}\right)^{\frac{3}{2}} \left[1 + \left(\frac{U}{R}\right)^{-3} U'^2 \right] (dx^4)^2 - \left(\frac{U}{R}\right)^{-\frac{3}{2}} U^2 d\Omega_4^2 \quad (4.3)$$

⁵ We write all the formulae for $N_f = 1$, but it is easy to generalize them to larger N_f .

where $U' = dU/dx^4$. The curve $U(x^4)$ is determined by solving the equations of motion that follow from the DBI action for the sixbrane

$$S_{D6} = -\tau_6 V_{1+1} V_4 R^{\frac{3}{2}} \int dx^4 U^{\frac{5}{2}} \sqrt{1 + \left(\frac{R}{U}\right)^3 U'^2} . \quad (4.4)$$

V_{1+1} is the volume of $(1+1)$ -dimensional Minkowski spacetime, V_4 is the volume of a unit S^4 , and $\tau_6 = T_6/g_s$ is the tension of a $D6$ -brane for constant dilaton $e^\Phi = g_s$.

Since the Lagrangian does not depend explicitly on x^4 , there is a first integral given by

$$\frac{U^{\frac{5}{2}}}{\sqrt{1 + \left(\frac{R}{U}\right)^3 U'^2}} = U_0^{\frac{5}{2}} \quad (4.5)$$

where U_0 is the value of U where $U' = 0$. We are looking for solutions which asymptotically approach a $D6$ and $\overline{D6}$ -brane a distance L apart, *i.e.* $U(x^4 \rightarrow \pm \frac{L}{2}) \rightarrow \infty$. U_0 is the smallest value of U along the brane.

There are two types of such solutions. The first is a straight brane and anti-brane stretched in U and located at $x^4 = \pm \frac{L}{2}$, corresponding⁶ to infinite U' and $U_0 = 0$ in (4.5). This solution preserves the chiral $SU(N_f)_L \times SU(N_f)_R$ symmetry.

The second is a symmetric U-shaped curve in the (U, x^4) plane. Integrating (4.5), it is given by

$$x^4(U) = \pm \int_{U_0}^U \frac{du}{\left(\frac{u}{R}\right)^{\frac{3}{2}} \left(\frac{u^5}{U_0^5} - 1\right)^{\frac{1}{2}}} = \pm \frac{1}{5} \frac{R^{\frac{3}{2}}}{U_0^{\frac{1}{2}}} \left[B\left(\frac{3}{5}, \frac{1}{2}\right) - B\left(\frac{U_0^5}{U^5}; \frac{3}{5}, \frac{1}{2}\right) \right] \quad (4.6)$$

where $B(z; a, b)$ is the incomplete Beta function, with $B(0; a, b) = 0$ and $B(1; a, b) = B(a, b)$ the usual Euler Beta function. The brane separation is

$$L = 2x^4(\infty) = \frac{2}{5} \frac{R^{\frac{3}{2}}}{U_0^{\frac{1}{2}}} B\left(\frac{3}{5}, \frac{1}{2}\right) . \quad (4.7)$$

This solution can be thought of as a $D6$ -brane and a $\overline{D6}$ -brane connected by a wormhole, whose radius at its narrowest point is U_0 . The fact that what looks asymptotically like $D6$

⁶ Since the curvature diverges as $U \rightarrow 0$, the small U behavior of this solution is not reliably described by supergravity. This is not going to matter for our considerations that will mainly involve the large U region.

and $\overline{D6}$ -branes is in fact part of a single connected curved brane implies that the chiral $U(N_f)_L \times U(N_f)_R$ symmetry is broken down to the diagonal $U(N_f)$.

To find the ground state of the system one has to compare the energies of the straight and curved solutions for fixed L . The energy difference $\Delta E \equiv E_{\text{straight}} - E_{\text{curved}}$ is proportional to

$$\Delta E \sim \int_0^{U_0} du (u - 0) + \int_{U_0}^{\infty} du u \left[1 - \left(1 - \frac{U_0^5}{u^5} \right)^{-\frac{1}{2}} \right] = -\frac{1}{5} U_0^2 B \left(-\frac{2}{5}, \frac{1}{2} \right) \approx 0.139 U_0^2, \quad (4.8)$$

so the curved configuration is preferred and chiral symmetry is broken. We see that the system is in the same phase for weak and strong coupling.

The regime of validity of the supergravity approximation is the same as in [2,1], $\lambda \gg L$ or large effective 't Hooft coupling, $g_s(U_0)N_c \gg 1$, [16].

4.2. Spectrum

The wormhole connecting the $D6$ and $\overline{D6}$ -branes in the vacuum configuration described in the previous subsection provides a mass to the fermions q_L, q_R . These correspond at strong coupling to fundamental strings that stretch from the bottom of the curved flavor branes down to $U = 0$.⁷ Using the metric (4.1) one finds that their mass is equal to $U_0/2\pi\alpha' = U_0/2\pi$, which from (4.7) is proportional to λ/L^2 . This should be compared to the weak coupling regime, where we saw that the dynamically generated mass of the fermions (3.17) was given by $m_f \sim (1/L)e^{-L/\lambda}$. Defining

$$m_f L = A(\lambda/L), \quad (4.9)$$

$A(x)$ behaves as

$$A(x) \simeq \begin{cases} e^{-1/x}, & x \rightarrow 0; \\ x, & x \rightarrow \infty. \end{cases} \quad (4.10)$$

⁷ The fact that the lowest lying state of such a string is a fermion can be understood as follows. The background (4.1) is close to flat except very close to $U = 0$. The open string in question ends on one side on a brane which is stretched in the directions (01), x^4 and an S^4 of radius U_0 . The other end of the string is at $U = 0$, where the curvature is large, but we can replace this region by boundary conditions corresponding to an open string ending on $D4$ -branes extended in (01234). Overall, the string has six Dirichlet-Neumann directions (the directions (23) and the S^4), so the lowest lying state is a (massive) fermion.

For $x \simeq 1$ both our approximations break down, but it is natural to expect $A(x)$ to be monotonic in x , so that the dynamically generated mass is a monotonic function of the coupling.

In addition to the long strings discussed above the spectrum includes light mesons which correspond to normalizable fluctuations of the scalars, gauge field and fermions on the $D6$ -brane. To study them we need to expand the DBI action around the background configuration (4.6). In the $D4 - D8 - \overline{D8}$ system it was shown in [2] that fluctuations of the gauge field give rise to massless Nambu-Goldstone (NG) bosons and massive vector mesons, while scalar fluctuations lead to a spectrum of massive scalar mesons. We will see that in our model gauge field fluctuations do not give rise to normalizable NG modes. Also, while the kinetic term of the gauge field gives rise to a massless vector, it acquires a mass by mixing with other modes via the Chern-Simons interaction (3.31).

We start with the action for the probe $D6$ -brane including the worldvolume gauge field:

$$S_{D6} = -T_6 \int d^7x e^{-\Phi} \sqrt{-\det(\tilde{g}_{AB} + 2\pi F_{AB})} + \frac{1}{8\pi^2} \int H_4 \wedge \omega_3(A) . \quad (4.11)$$

Here H_4 is the RR field sourced by the color $D4$ -branes (as in (3.31)) and ω_3 is the Chern-Simons three-form (3.32). In the four-dimensional model of [2] there is an analogous coupling to the Chern-Simons five-form, which plays an important role in anomaly cancellation, but since it is cubic in the gauge field, it does not affect the mass spectrum. In our case, the Chern-Simons term contains terms quadratic in the gauge field and must be included in the analysis.

We focus on $SO(5)$ singlet gauge field fluctuations that are independent of the S^4 coordinates. Denoting the remaining components of the gauge field by A_m , $m = 0, 1, U$ and expanding the action to quadratic order we find

$$S_{D6}^{(\text{quad})} = -T_6 V_4 (2\pi)^2 \int d^2x dU e^{-\Phi} \sqrt{-\det \tilde{g}} \frac{1}{4} F_{mn} F^{mn} + \frac{N_c}{4\pi} \int d^2x dU \epsilon^{mnp} A_m F_{np} . \quad (4.12)$$

In this expression $\det \tilde{g}$ is the determinant of the induced metric on the $D6$ -brane (4.3), and the indices m, n are raised and lowered with the m, n components of this metric. Using the solution (4.6) we can rewrite the action (4.12) as

$$S_{D6}^{(\text{quad})} = -T_6 V_4 (2\pi)^2 \int d^2x dU \left[\frac{1}{4} G(U) F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} H(U) F^\mu_U F_{\mu U} \right] \\ + \frac{N_c}{4\pi} \int d^2x dU \epsilon^{\mu\nu} (A_U F_{\mu\nu} + 2A_\mu F_{\nu U}) . \quad (4.13)$$

In (4.13) the indices $\mu, \nu = 0, 1$ are raised and lowered with the flat metric $\eta_{\mu\nu}$ and all U dependence is contained in the functions

$$\begin{aligned} G(U) &= \frac{R^6 \gamma(U)^{\frac{1}{2}}}{g_s U^2} , \\ H(U) &= \frac{R^3 U}{g_s \gamma(U)^{\frac{1}{2}}} , \\ \gamma(U) &= \frac{U^5}{U^5 - U_0^5} . \end{aligned} \tag{4.14}$$

We now separate variables and expand the gauge field in modes in the U direction as follows (with normalization conditions to be determined shortly)

$$\begin{aligned} A_\mu(x^\nu, U) &= \sum_n B_\mu^{(n)}(x^\nu) \psi_n(U) , \\ A_U(x^\nu, U) &= \sum_n \pi^{(n)}(x^\nu) \phi_n(U) . \end{aligned} \tag{4.15}$$

Defining $B_{\mu\nu}^{(n)} = \partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)}$ this leads to the action

$$\begin{aligned} S &= -T_6 V_4 (2\pi)^2 \int d^2 x dU \left[\sum_{n,m} G(U) \psi_n \psi_m \frac{1}{4} B_{\mu\nu}^{(n)} B^{(m)\mu\nu} \right. \\ &\quad \left. + \frac{1}{2} H(U) \sum_{n,m} \left(\partial_\mu \pi^{(n)} \phi_n - B_\mu^{(n)} \partial_U \psi_n \right) \left(\partial^\mu \pi^{(m)} \phi_m - B^{(m)\mu} \partial_U \psi_m \right) \right] \\ &\quad + \frac{N_c}{4\pi} \int d^2 x dU \left[\epsilon^{\mu\nu} \sum_{n,m} \left(\pi^{(n)} \phi_n B_{\mu\nu}^{(m)} \psi_m + 2 B_\mu^{(n)} \psi_n \left(\partial_\nu \pi^{(m)} \phi_m - B_\nu^{(m)} \partial_U \psi_m \right) \right) \right] . \end{aligned} \tag{4.16}$$

We see from the first line of (4.16) that the kinetic term for $B_\mu^{(n)}$ is canonical provided that the wavefunctions $\psi_m(U)$ satisfy the normalization condition

$$T_6 V_4 (2\pi)^2 \int dU G(U) \psi_n(U) \psi_m(U) = \delta_{n,m} . \tag{4.17}$$

Terms on the second line of (4.16) give diagonal and canonically normalized mass terms for $B_\mu^{(n)}$ with mass squared m_n^2 provided that we also have

$$T_6 V_4 (2\pi)^2 \int dU H(U) \partial_U \psi_n(U) \partial_U \psi_m(U) = m_n^2 \delta_{n,m} . \tag{4.18}$$

Equations (4.17) and (4.18) are compatible if ψ_n satisfies the eigenvalue equation

$$\frac{1}{G(U)} \partial_U [H(U) \partial_U \psi_n(U)] = -m_n^2 \psi_n(U) . \tag{4.19}$$

Finally, in order to have canonically normalized kinetic energy terms for the $\pi^{(n)}$ we require

$$T_6 V_4 (2\pi)^2 \int dU H(U) \phi_n(U) \phi_m(U) = \delta_{n,m} . \quad (4.20)$$

This can be satisfied by choosing $\phi_n = \partial_U \psi_n / |m_n|$ for modes with $m_n \neq 0$. For $m_n = 0$, a formal extrapolation of the above analysis leads to a zero mode, $\phi_0 \sim \partial_U \psi_0 \sim 1/H(U)$. In the $D4 - D8 - \overline{D8}$ system, an analogous mode was shown in [2] to be the massless Nambu-Goldstone boson associated with the breaking of $U(N_f)_L \times U(N_f)_R$ to $U(N_f)_{\text{diag}}$. In our case, in contrast to [2], this putative zero mode is neither orthogonal to the other ϕ_n , nor normalizable. The latter follows from the fact that $H(U) \sim U$ at large U so that the integral

$$\int dU H(U) \phi_0(U) \phi_0(U) \sim \int \frac{dU}{H(U)} \quad (4.21)$$

diverges logarithmically at large U . Thus, there is no normalizable zero mode and hence no massless NG mode in the spectrum. This seems odd, since we have established above that the symmetry is broken and one expects this to lead to a NG boson on general grounds. We will discuss the resolution of this puzzle below.

Another puzzling feature of the spectrum is the existence of a normalizable zero mode for the gauge field, corresponding to $\psi_0 = \text{const}$ in (4.15). It satisfies (4.19) with $m_n = 0$ and from (4.17) is normalizable since $G(U) \sim 1/U^2$ at large U . This seems to suggest that the $U(N_f)$ symmetry on the $D6$ -branes is gauged in $1+1$ dimensions, which is unexpected from the weak coupling point of view. We will see below that this mode is not an eigenstate of the full quadratic action (4.16).

To summarize, in the expansion (4.15) there is a normalizable zero mode ψ_0 in the expansion of A_μ , but the expansion of A_U involves a sum only over non-zero modes for which $\phi_n = \partial_U \psi_n / |m_n|$. We thus see that A_U is pure gauge, $A_U = \partial_U \Lambda$ with $\Lambda = \sum_n \frac{1}{|m_n|} \pi^{(n)}(x) \psi_n(U)$. Making a gauge transformation by Λ to set $A_U = 0$ and substituting (4.15) into the action (4.13) leaves us with the two-dimensional action

$$S_{D6} = - \int d^2x \left[\frac{1}{4} B_{\mu\nu}^{(0)} B^{\mu\nu(0)} + \sum_{n \geq 1} \left(\frac{1}{4} B_{\mu\nu}^{(n)} B^{\mu\nu(n)} + \frac{1}{2} m_n^2 B_\mu^{(n)} B^{\mu(n)} \right) \right] \\ - \int d^2x \sum_{m,n} \epsilon^{\mu\nu} \mathcal{M}_{mn}^2 B_\mu^{(n)} B_\nu^{(m)} \quad (4.22)$$

where

$$\mathcal{M}_{nm}^2 = -\mathcal{M}_{mn}^2 = \frac{N_c}{2\pi} \int dU \psi_n(U) \partial_U \psi_m(U) . \quad (4.23)$$

It is clear from (4.22) that the fields $B_\mu^{(n)}$ are not mass eigenstates because of the off-diagonal mixing. It is difficult to compute the mass eigenstates explicitly since to do so we would have to diagonalize the infinite-dimensional matrix with diagonal components m_n^2 and off diagonal components $\mathcal{M}_{n,m}^2$. However, there is no reason why the determinant of this mass matrix should vanish, so this process presumably leaves us with mass eigenstates all with non-zero masses. In particular, the massless gauge field $B_\mu^{(0)}$ mixes with the massive modes via (4.23) with $n = 0$

$$\mathcal{M}_{0m}^2 \propto \frac{N_c}{2\pi} \int dU \partial_U \psi_M(U) . \quad (4.24)$$

It is easy to check that normalizable solutions of (4.19) approach a non-zero constant as $U \rightarrow \infty$, so the integral in (4.24), and hence the mixing of $B_\mu^{(0)}$ with the other modes, is non-zero.

One general observation that can be made about the spectrum is the following. It was recently emphasized in [17] that the holographic distance-energy relation of [18] coupled with the consistency of holography for open and closed string modes implies a universal scaling of meson masses in the strongly coupled, supergravity regime:

$$M_{\text{meson}} \sim \frac{m_f}{g_{\text{eff}}(m_f)} \quad (4.25)$$

where m_f is the fermion mass and g_{eff}^2 is the effective 't Hooft coupling evaluated at the scale m_f . In our model it is given by

$$g_{\text{eff}}^2(m_f) \simeq g_5^2 N_c m_f \sim \lambda m_f . \quad (4.26)$$

Using the fact that $m_f \sim \lambda/L^2$, (4.25) leads to the prediction that all meson masses are proportional to $M_{\text{meson}} \sim 1/L$ and are thus deeply bound since $M_{\text{meson}} \ll m_f$. To see that this is in fact true, note that the mass parameters m_n^2 in (4.22) are determined by the eigenvalue equation (4.19). Writing it in terms of $\bar{U} = U/U_0$ gives

$$\bar{U}^2 \gamma^{-\frac{1}{2}} \partial_{\bar{U}} \left(\bar{U} \gamma^{-\frac{1}{2}} \partial_{\bar{U}} \psi_n \right) = -\frac{R^3}{U_0} m_n^2 \psi_n . \quad (4.27)$$

Since all the dependence on the parameters λ and L is now contained in the factor $R^3/U_0 \sim L^2$, we see that the eigenvalues m_n^2 are all proportional to $1/L^2$ and independent of the 't Hooft coupling λ .

The mixing parameters \mathcal{M}_{mn}^2 (4.23) also have this property. The normalization condition for ψ_n (4.17) can be written as

$$\int \frac{d\bar{U}}{\bar{U}^2} \gamma^{\frac{1}{2}} \bar{\psi}_n \bar{\psi}_m = \delta_{n,m} \quad (4.28)$$

where

$$\psi_n = \frac{c}{L\sqrt{N_c}} \bar{\psi}_n \quad (4.29)$$

for c a constant, independent of L and λ . Therefore, $\mathcal{M}_{m,n}^2$ (4.23) also scale like $1/L^2$.

To summarize, fluctuations of the gauge field lead to a spectrum of massive vector fields in $1+1$ dimensions. The masses all scale like $1/L$ and are determined by solving the eigenvalue equation (4.19) and then diagonalizing the quadratic form in (4.22). A similar analysis leads to a spectrum of massive scalar and fermion fields with masses that scale like $1/L$.

In contrast to the model of Sakai and Sugimoto [2], we do not find any massless NG bosons in the spectrum, in spite of the apparent breaking of $U(N_f)_L \times U(N_f)_R$ to the diagonal $U(N_f)$. The resolution of this puzzle was pointed out in a closely related context in [8]. We saw in section 3 (after eq. (3.34)) that in the weak coupling limit, the NG bosons are displaced from the intersection of the branes by an amount of order $\lambda^{1/3}$ in the U direction. Extrapolating this to strong coupling we find that the $U(N_f)$ degrees of freedom live in the transition region between the near-horizon geometry of the $D4$ -branes and the asymptotically flat space far from the branes. Therefore, they are not visible in an analysis that restricts attention to the near-horizon geometry. Note that this kind of behavior is only possible in two dimensions where the NG bosons are decoupled from the massive physics. In higher dimensions similar behavior is familiar from the analysis of the center of mass modes in brane systems, which decouple in the near-horizon limit.

The scalar, fermion and vector mesons obtained by studying small fluctuations of the D -branes are quite different from the bound states of the GN model whose masses are given by (3.35). In particular, as is clear from our analysis, at weak coupling the color degrees of freedom living on the $D4$ -branes are essentially non-dynamical, and their main effect is to produce the small attractive force between the fermions q_L, q_R . All the states (3.35) can then be thought of as bound states of these fermions.

On the other hand, at strong coupling, the degrees of freedom living on the $D4$ -branes (or, more properly, compactified $M5$ -branes) are strongly interacting, and the states with masses of order $1/L$ are best thought of as containing the fermions $q_{L,R}$ as well as the

color degrees of freedom. They have the qualitative form $q^\dagger \Phi^n q$, where q stands for the fermions living at the intersection, and Φ stands for all the bosonic and fermionic degrees of freedom on the $D4$ -branes. Therefore, these states do not have a direct analog at weak coupling. It is interesting that they are much lighter than the fermions themselves, like Nambu-Goldstone bosons of a spontaneously broken symmetry.

To study the analogs of the GN states (3.35) at strong coupling, one needs to consider long fundamental strings stretched between the curved $D6$ -branes and $U = 0$. As mentioned above, a single string of this sort is a massive fermion. A few such strings might form bound states, like their weak coupling counterparts. This is an interesting problem for future study.

5. Finite temperature

At zero temperature, the intersecting brane model of sections 2 – 4 exhibits dynamical chiral symmetry breaking both at weak coupling (discussed in section 3), where it reduces to the Gross-Neveu model, and at strong coupling, where it is well described by probe $D6$ -brane dynamics in the near-horizon geometry of $D4$ -branes (section 4). In this section we will study this model at finite temperature, and in particular analyze the finite temperature phase transition at which chiral symmetry is restored. In the following two subsections we do that in turn for weak and strong coupling. In the last subsection we briefly address the interpolation between the two regimes.

5.1. The Gross-Neveu model at finite temperature

The finite temperature properties of the Gross-Neveu model were studied in [19-21] and many subsequent papers. Here we briefly review the results, which are relevant for our intersecting D-brane system in the limit $\lambda_{gn} \rightarrow 0$, $E \sim m_f$ discussed in section 3. In this limit the D-brane system is described by the local Lagrangian (3.4), which can be written in a way similar to (3.7):

$$\mathcal{S}_{\text{eff}} = \int d^2x \left(i q_L^\dagger \bar{\sigma}^\mu \partial_\mu q_L + i q_R^\dagger \sigma^\mu \partial_\mu q_R + \sigma q_R^\dagger \cdot q_L + \bar{\sigma} q_L^\dagger \cdot q_R - \frac{N_c}{\lambda_{gn}} \sigma \bar{\sigma} \right). \quad (5.1)$$

The expectation value of σ is the mass of the fermions q_L, q_R . To calculate it, it is convenient to integrate out the fermions and minimize the resulting effective potential for σ .

For zero temperature, the effective potential takes the form [5]

$$V_{\text{eff}} = \frac{1}{\lambda_{gn}} \sigma \bar{\sigma} - \int \frac{d^2 k}{(2\pi)^2} \ln \left(1 + \frac{\sigma \bar{\sigma}}{k^2} \right) . \quad (5.2)$$

Stationary points of V_{eff} satisfy

$$\frac{\delta V_{\text{eff}}}{\delta \bar{\sigma}} = \frac{\sigma}{\lambda_{gn}} - \int \frac{d^2 k}{(2\pi)^2} \frac{\sigma}{k^2 + \sigma \bar{\sigma}} = 0 , \quad (5.3)$$

which is equivalent to the gap equation (3.15) discussed in section 3 in the local, GN, limit. $\sigma = 0$ is a local maximum of the potential (5.2); the minimum corresponds to the non-trivial solution of (5.3) where chiral symmetry is broken.

To study chiral symmetry breaking at finite temperature, we need to evaluate the expectation value

$$\langle \sigma \rangle = \frac{\text{Tr} \sigma e^{-\beta H}}{\text{Tr} e^{-\beta H}} \quad (5.4)$$

where H is the Hamiltonian and $1/\beta = T$ the temperature. This can be done by studying the theory corresponding to (5.1) on $\mathbb{R} \times S^1$, with the Euclidean time living on a circle of circumference β with anti-periodic boundary conditions for the fermions q_L, q_R . This amounts to replacing in (5.2), (5.3),

$$\int \frac{dk_0}{2\pi} \rightarrow \frac{1}{\beta} \sum_{r \in Z + \frac{1}{2}} \quad (5.5)$$

with $k_0 = 2\pi r/\beta$ the momentum in the Euclidean time direction. Making this replacement in (5.2) and minimizing the effective potential with respect to σ gives the leading contribution to the free energy $F(\beta)$. The value of σ at the minimum gives the expectation value (5.4).

The stationarity condition of V_{eff} is now

$$\frac{\delta V_{\text{eff}}}{\delta \bar{\sigma}} = \frac{\sigma}{\lambda_{gn}} \left[1 - \frac{\lambda_{gn}}{\beta} \sum_{r \in Z + \frac{1}{2}} \int_{-\Lambda}^{\Lambda} \frac{dp}{2\pi} \frac{1}{\left(\frac{2\pi r}{\beta} \right)^2 + p^2 + |\sigma|^2} \right] \quad (5.6)$$

Following [21] one can replace the sum over r by a contour integral over energy,

$$\frac{1}{\beta} \sum_{r \in Z + \frac{1}{2}} \rightarrow \oint \frac{d\epsilon}{2\pi i} \frac{1}{e^{\beta\epsilon} + 1} , \quad (5.7)$$

where the contour surrounds the poles at $\beta\epsilon = 2\pi ri$. Deforming the contour and picking up the residues of the poles of the r.h.s. of (5.6) at

$$\epsilon = \pm\epsilon_p = \pm\sqrt{p^2 + |\sigma|^2} \quad (5.8)$$

gives

$$\frac{\delta V_{\text{eff}}}{\delta\sigma} = \frac{\sigma}{\lambda_{gn}} \left[1 - \frac{\lambda_{gn}}{4\pi} \int_{-\Lambda}^{\Lambda} \frac{dp}{\epsilon_p} \tanh\left(\frac{1}{2}\beta\epsilon_p\right) \right]. \quad (5.9)$$

The stationary points are $\sigma = 0$ and the solution of

$$1 = \frac{\lambda_{gn}}{2\pi} \int_0^{\Lambda} \frac{dp}{\epsilon_p} \tanh\left(\frac{1}{2}\beta\epsilon_p\right). \quad (5.10)$$

As $\beta \rightarrow \infty$ we can approximate $\tanh \frac{1}{2}\beta\epsilon_p \simeq 1$, and (5.10) (with $\sigma(\beta \rightarrow \infty) = m_f$) reads

$$1 = \frac{\lambda_{gn}}{2\pi} \int_0^{\Lambda} \frac{dp}{\epsilon_p} \simeq \frac{\lambda_{gn}}{2\pi} \ln \frac{2\Lambda}{m_f}. \quad (5.11)$$

Thus, the mass of the fermions at zero temperature is given by

$$m_f = 2\Lambda e^{-\frac{2\pi}{\lambda_{gn}}}, \quad (5.12)$$

which agrees with (3.17) after allowing for a different definition of the cutoff Λ .

For low temperature, *i.e.* large but finite β , $\sigma = 0$ is still a local maximum. Indeed, for $\sigma = 0$, $\epsilon_p = |p|$ and the integral in (5.9) is given by [21]:

$$\int_0^{\Lambda} \frac{dp}{p} \tanh\left(\frac{1}{2}\beta p\right) \simeq \ln(1.14\beta\Lambda), \quad (5.13)$$

where we assumed that $\beta\Lambda \gg 1$, as appropriate in the local GN limit. Thus, one has

$$\frac{\delta^2 V_{\text{eff}}(\sigma = 0)}{\delta\sigma\delta\sigma} = \frac{1}{\lambda_{gn}} \left(1 - \frac{\lambda_{gn}}{2\pi} \ln(1.14\beta\Lambda) \right) = -\frac{1}{2\pi} \ln(0.57\beta m_f), \quad (5.14)$$

where in the last step we used (5.11). We see that $\sigma = 0$ is a maximum of V_{eff} for $\beta > \beta_c$ and a minimum for $\beta < \beta_c$. The critical temperature is

$$T_c = \frac{1}{\beta_c} = 0.57m_f. \quad (5.15)$$

Below that temperature, the minimum of the effective potential is at a non-zero value of σ , $\sigma = m_f(\beta)$, which corresponds to a non-trivial solution of (5.10) with chiral symmetry

broken. As $T \rightarrow T_c$, $m_f(\beta) \rightarrow 0$; for $T > T_c$ the only solution of (5.10) is $\sigma = 0$, and chiral symmetry is restored. The phase transition at $T = T_c$ is second order since the order parameter $m_f(\beta)$ is continuous at the transition.

It should be noted that, as in section 3, the discussion above is strictly speaking valid only for infinite N_c . For finite N_c the order parameter (5.4) is not really constant in space as was assumed in our analysis. Averaging over sufficiently large distances, this expectation value vanishes for all T . However the domains over which it is roughly constant grow with N_c . Thus, the large N_c analysis is a useful guide to the behavior of the system for finite N_c as well (see [19-21] for further discussion).

5.2. Strong coupling analysis at finite temperature

At strong coupling the dynamics of the intersecting branes is described by studying probe $D6$ -branes in the near-horizon geometry of the N_c $D4$ -branes. To study the system at finite temperature we compactify Euclidean time on a circle of circumference β , with anti-periodic boundary conditions for the fermions. The resulting metric and dilaton are given by

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} (f(U)(dx^0)^2 + (dx^1)^2 + (dx^4)^2) + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{1}{f(U)}dU^2 + U^2 d\Omega_4^2\right),$$

$$e^\Phi = g_s \left(\frac{U}{R}\right)^{\frac{3}{4}},$$
(5.16)

with

$$f(U) = 1 - \frac{U_T^3}{U^3}.$$
(5.17)

This geometry is a continuation to Euclidean space of the non-extremal $D4$ -brane background. The non-extremality parameter U_T is related to the temperature via

$$\beta = \frac{4\pi R^{\frac{3}{2}}}{3U_T^{\frac{1}{2}}}.$$
(5.18)

For zero temperature one has $U_T = 0$, and the solution (5.16), (5.17) reduces to (4.1).

To calculate the free energy of the intersecting brane theory we need to solve for the shape of the $D6$ -branes in the geometry (5.16). This can be done similarly to the closely related $D4 - D8 - \overline{D8}$ case which was studied in [22,23].

The DBI action for the sixbranes is

$$S_{D6} = -T_6 V_{1+1} V_4 R^{\frac{3}{2}} \int dx^4 U^{\frac{5}{2}} \sqrt{f(U) + \left(\frac{R}{U}\right)^3 U'^2} . \quad (5.19)$$

The generalization of (4.5) to non-zero temperature is

$$\frac{U^{\frac{5}{2}} \sqrt{f(U)}}{\sqrt{1 + \left(\frac{R}{U}\right)^3 \frac{U'^2}{f(U)}}} = U_0^{\frac{5}{2}} \sqrt{f(U_0)} . \quad (5.20)$$

U_0 is the lowest value of U reached by the brane.

We need to solve (5.20) for the shape of the branes $U(x^4)$, as in section 4. There are again two possible solutions. In one the $D6$ and $\overline{D6}$ -branes wrap the smooth space labeled by (U, x^0) , at two different values of x^4 a distance L apart. In this configuration the branes extend all the way to the Euclidean horizon; thus $U_0 = U_T$. The chiral $U(N_f)_L \times U(N_f)_R$ symmetry (2.3) is preserved.

In the other solution, the $D6$ and $\overline{D6}$ -branes are connected, with $U = U(x^4)$, and $U_0 > U_T$. This solution breaks the chiral $U(N_f)_L \times U(N_f)_R$ symmetry to the diagonal $U(N_f)$.

To see which of the solutions has lower free energy we need to compare the values of the corresponding effective potentials

$$\begin{aligned} \Delta E = E_{\text{straight}} - E_{\text{curved}} &\sim \int_{U_T}^{U_0} dU (U - 0) + \int_{U_0}^{\infty} dU U \left[1 - \left(1 - \frac{U_0^5 f(U_0)}{U^5 f(U)} \right)^{-\frac{1}{2}} \right] \\ &\sim \int_{y_T}^1 dy (y - 0) + \int_1^{\infty} dy y \left[1 - \left(1 - \frac{f(1)}{y^5 f(y)} \right)^{-\frac{1}{2}} \right] , \end{aligned} \quad (5.21)$$

where in the last line we introduced the dimensionless coordinate $y = U/U_0$.

The numerical result for $\Delta E(T)$ is plotted in figure 2. For low temperature, the curved solution is energetically preferred. ΔE vanishes at a critical temperature corresponding to $y_T^c = U_T^c/U_0 \approx 0.613$, above which the symmetric solution becomes energetically preferred. Thus, the system exhibits a first order phase transition at a temperature $T_c \simeq 1/L$. The constant of proportionality can be fixed as follows. In the low temperature phase, L , U_T and U_0 are related by

$$L = 2 \frac{R^{\frac{3}{2}}}{U_0^{\frac{1}{2}}} \int_1^{\infty} dy \frac{y^{-\frac{3}{2}}}{\sqrt{f(y)} \sqrt{\frac{f(y)}{f(1)} y^5 - 1}} . \quad (5.22)$$

At the critical temperature $y_T = y_T^c$ one finds $L \approx 1.097 \frac{R^{3/2}}{U_0^{1/2}}$, which implies

$$T_c \approx 0.205/L . \quad (5.23)$$

The phase transition at T_c is first order since the fermion mass jumps from a non-zero value to zero as T crosses T_c . The mass right below the transition is equal to the energy of a string stretched from the curved $D6$ -branes to the Euclidean horizon at $U = U_T$,

$$m_f = \frac{1}{2\pi} \int_{U_T}^{U_0} \sqrt{-g_{00}g_{UU}} dU = \frac{1}{2\pi} (U_0 - U_T) . \quad (5.24)$$

Thus $\frac{m_f(T_c)}{m_f(0)} = 1 - y_T^c \approx 0.387$.

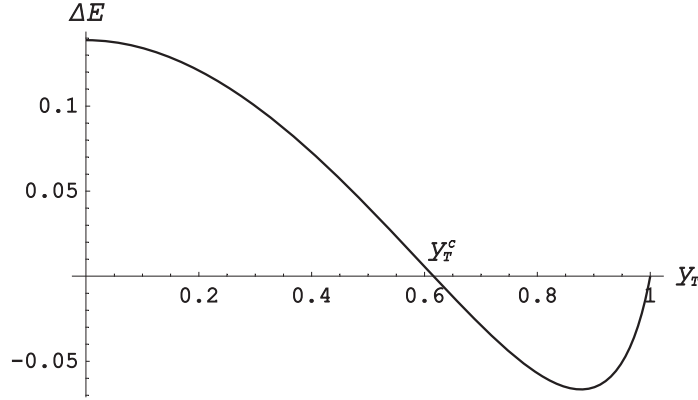


Fig. 2: Difference between the free energy of the straight and curved solutions as a function of temperature.

5.3. Interpolating between weak and strong coupling

In the last two subsections we discussed the finite temperature phase transition in the limits $\lambda/L \rightarrow 0$ and $\lambda/L \rightarrow \infty$. It would be interesting to understand the interpolation between the two regimes.

The transition temperature, T_c , is given in the two limits by (5.15) and (5.23), respectively. A natural interpolation to all values of the coupling is

$$\frac{T_c}{m_f} = H(m_f L) . \quad (5.25)$$

Here $m_f L$ is a measure of the coupling λ/L . It goes to zero in the GN limit and to infinity in the supergravity one (see (4.9), (4.10)). H is a function to be determined. Eq. (5.15) implies that

$$H(0) = 0.57 \quad (5.26)$$

while (5.23) implies that

$$H(x \rightarrow \infty) \simeq \frac{0.205}{x} . \quad (5.27)$$

It is natural to expect that $H(x)$ varies smoothly and monotonically with x .

Another interesting observable is the order parameter right below the transition,

$$\frac{m_f(T_c)}{m_f(0)} = B(m_f L) . \quad (5.28)$$

From the GN analysis we know that

$$B(0) = 0 , \quad (5.29)$$

i.e. the symmetry restoration transition is second order. We expect the transition to remain second order for sufficiently small λ/L , assuming that the physics at finite λ/L is continuously connected to that of the GN model.⁸ The reason is that the effective potential (5.9) behaves near the origin like

$$V_{\text{eff}} = a_1 |\sigma|^2 + a_2 |\sigma|^4 + \dots \quad (5.30)$$

where a_1 is given by (5.14) and changes sign at $T = T_c$ and a_2 is strictly positive. Small λ/L corrections to a_1 and a_2 cannot change the fact that the transition happens at the point where a_1 changes sign and is second order.

At strong coupling we found

$$B(\infty) = 0.387 . \quad (5.31)$$

We expect B to vanish for λ/L below some critical value, and then smoothly interpolate between zero and (5.31) as $\lambda/L \rightarrow \infty$.

6. Discussion

In this paper we analyzed a model of intersecting $D4$ and $D6$ -branes and antibranes that has the interesting property that the chiral symmetry of the original brane configuration (2.3) is broken to the diagonal subgroup by the dynamics. The model contains an adjustable parameter, λ/L , which governs the strength of the coupling between left and

⁸ We thank O. Aharony for a discussion of this issue.

right-moving fermions living at different intersections. For weak coupling it reduces to the Gross-Neveu model [5], a vector model which is exactly solvable⁹ at large N_c . For strong coupling it can be analyzed by studying the DBI action for $D6$ -branes in the near-horizon geometry of N_c $D4$ -branes and again breaks chiral symmetry.

As one would expect, the strength of chiral symmetry breaking increases with the coupling. For $\lambda/L \ll 1$, the dynamically generated fermion mass m_f (3.17) is much smaller than the natural scales in the problem, $1/L$ and $1/\lambda$. Indeed, $m_f L$, $m_f \lambda$ go to zero as the coupling goes to zero. For $\lambda/L \gg 1$ the fermion mass is of order λ/L^2 and is much larger than the other scales: $m_f L$, $m_f \lambda$ go to infinity as the coupling goes to infinity (see (4.9), (4.10)). The strong interactions between the fermions and the degrees of freedom living on the $D4$ -branes give rise to strongly bound mesons whose masses are much smaller than m_f (their binding energy is thus almost 100%). These bound states can be studied by expanding the DBI action of the sixbranes around the vacuum solution.

Another manifestation of the dependence of the strength of chiral symmetry breaking on the coupling is the thermodynamic properties described in section 5. For strong coupling, the transition is strongly first order – the fermion mass right below the transition is of order m_f (see (5.24)), which as mentioned above is large in natural units. As the strength of the interaction decreases the transition becomes weaker and it is second order at weak coupling (as described in subsection 5.1).

There are many possible directions for further investigation of this and related models. The Gross-Neveu model has a spectrum of meson excitations with masses given by (3.35) whose S-matrix is known due to integrability. In our setting, the GN model is obtained in the limit $\lambda/L \rightarrow 0$ described in section 3. It would be interesting to see whether the integrability persists for finite λ/L , and if so what one can learn about the model by using it. In particular it would be interesting to determine the mass spectrum, S-matrix and the thermodynamic functions H (5.25) and B (5.28) for all values of the coupling.

Another issue involves the stability of the system beyond the large N_c limit. In the full IIA string theory the brane configuration of figure 1 is unstable. The $D6$ and $\overline{D6}$ -branes attract each other by massless closed string exchange and will start moving towards each other if placed in this configuration. These gravitational effects turn off in the limit $g_s \rightarrow 0$, and therefore can be neglected in the 't Hooft limit that we took. It is natural to ask what happens for finite N_c . We suspect that the system is still stable although this deserves

⁹ In fact this model is integrable at finite N_c as well.

more careful study. On the supergravity side, taking the near-horizon limit means that the $D6$ and $\overline{D6}$ -branes become infinitely separated near the boundary at $U = \infty$. Therefore we expect the force per unit worldvolume to go to zero, while the mass per unit worldvolume remains constant. On the field theory side the dynamics is governed by the $(2, 0)$ theory compactified on a circle. The $D6$ -branes are interpreted as defects in this theory and we see no clear reason why the introduction of defects should result in a drastic instability of the theory.

There are many natural generalizations of our model. The brane configuration of figure 1 contains a single stack of $D6$ -branes all at the same point in the \mathbb{R}^3 labeled by (234), and N_f antibranes at a distance L in \mathbb{R}^3 . As a consequence, the dynamics depends on a single parameter, λ/L . One could generalize this construction by putting the $D6$ -branes and antibranes at arbitrary points in \mathbb{R}^3 , and studying the dynamics as a function of their relative positions. At weak coupling this gives a generalization of the GN model, while at strong coupling it gives a generalization of the DBI dynamics of section 4. In both limits there is a rich phase structure with different patterns of symmetry breaking depending on the positions of the branes and antibranes in \mathbb{R}^3 . It would be interesting to analyze it.

Another generalization involves making the $SU(N_c)$ gauge fields dynamical. This can be done by compactifying the directions (234) on a torus of volume V_{234} . The resulting model is very similar to the 't Hooft model of two-dimensional QCD [24]. The two-dimensional 't Hooft coupling, which has units of energy squared and sets the scale of the masses of mesons, is given by

$$\lambda_2 = \frac{\lambda}{V_{234}} . \quad (6.1)$$

The dynamics depends on the hierarchy of scales and the strength of the various couplings. At weak coupling and large volume V_{234} , the scale of confinement $\sqrt{\lambda_2}$ is much smaller than that of chiral symmetry breaking m_f (3.17). Thus, at the scale of chiral symmetry breaking one can neglect the dynamics of the two-dimensional gauge field, and the mass generation of the fermions occurs as in section 3. At the much lower energy scale $\sqrt{\lambda_2}$ the effect of confinement kicks in, and the analysis of [24] is applicable (with fermion mass m_f). As the volume V_{234} decreases the physics changes, and for $\lambda_2 \gg m_f^2$ the gauge dynamics becomes strong at energy scales at which the fermions are still massless.

Thus, in this model confinement and chiral symmetry breaking are distinct phenomena and can occur at widely separated energies. This property was noted in a four-dimensional intersecting brane system in [1] and is a general feature of such systems.

Other brane configurations with qualitatively similar dynamics are obtained by changing the dimensionalities of the color and flavor branes and that of the intersection. This leads to many interesting models, including that of [2,1], one that was considered recently in [25], and many others. We will discuss some features of such models in a companion paper [26].

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